Searching for Exactly Solvable Models of Active Matter

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Funder: EPSRC grant No. EP/R513209/1

Abstract
Active matter is a type of non-equilibrium system in which particles generate their own motion. Simple models of active matter have been used to model a wide range of systems, primarily in the context of biophysics. Unlike equilibrium systems, there is no universal framework for describing non-equilibrium behaviour, so it is helpful to find examples of systems that can be solved exactly to understand the underlying universal principles. In this project, we carry out a systematic search for models which are solvable by the Bethe Ansatz (BA), one of the most powerful approaches for obtaining exact solutions of 1D systems.

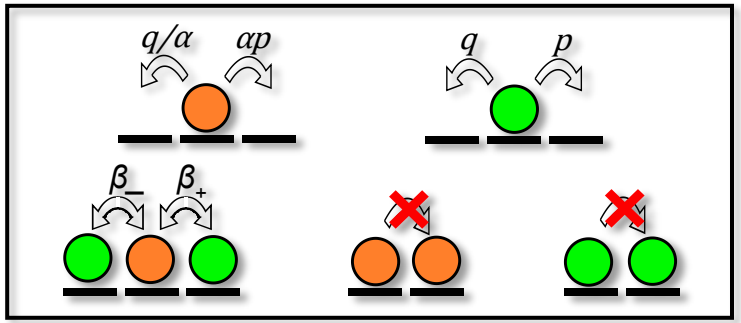

Fig. 1: Dynamics in a minimal driven interacting particle system

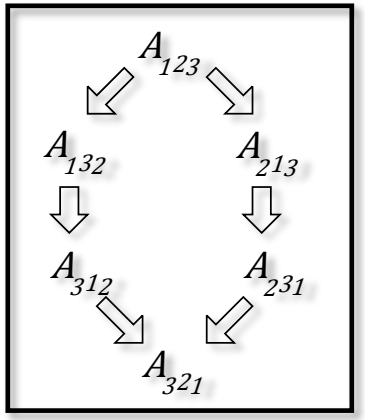

Fig. 2: Schematic representation of Yang-Baxter relations

Project Description
We consider a minimal driven (active), interacting model with two classes of particles in 1D (Fig. 1). First-class particles hop right and left with rates αp and q/α and may overtake second-class particles right and left with rates β_+ and β_-. Second-class particles hop right and left with rates p and q. All particles interact via hard-core repulsion. The interactions generate a non-equilibrium steady state with non-trivial probability distributions.

Special cases of this problem have been studied in the past using the BA. A necessary condition for the BA to work is given by the Yang-Baxter equations (Fig. 2). In this project, we consider a general form of this model and use Mathematica to find restrictions on the parameters which ensure that the Yang-Baxter equations are satisfied, giving us a list of solvable models.

Results
- We carried out a systematic search for two species driven, interactive models solvable by BA.
- We recovered many known cases and one previously unstudied system.
- This will be the first example of a partially asymmetric system with particles with different hopping rates that is solved using the BA.
- Similar classifications have been carried out for different categories of BA solvable models. This result is important for completing this classification and for narrowing the directions of future research.

Further reading
- Golinelli, O., & Mallick, K. (2006). The asymmetric simple exclusion process: an integrable model for non-equilibrium statistical mechanics. *Journal of Physics A: Mathematical and General*, *39*(41), 12679.
- Derrida, B., & Evans, M. R. (1999). Bethe ansatz solution for a defect particle in the asymmetric exclusion process. *Journal of Physics A: Mathematical and General*, *32*(26), 4833.